CMPS 251-Numerical Computing
Assignment 1
Due Monday, September 21, 2015

Reading Material:
- Cheney & Kincaid: Sections 1.1 and 1.2
- Matlab tutorial

Notes: You are encouraged in work individually on the assignment. Piazza can be used to ask questions (without requesting a solution !!).

Problem 1 *Horner's Algorithm*

Given a polynomial p of degree n and an input root r, write a MATLAB program that uses the Horner's algorithm to evaluate the polynomial p and its derivative at r, that is, $p(r)$ and $p'(r)$, respectively. Use the pseudocode in page 11 from the textbook (Slide 38 of Section 1.1 posted on Moodle).
Test your function for the following polynomials. Additionally, show the detailed calculations by hand.

1. $2x^4 + 9x^2 - 16x + 12$ at -6
2. $2x^4 - 3x^3 - 5x^2 + 3x + 8$ at 2
3. $3x^5 - 38x^3 + 5x^2 - 1$ at 4

Problem 2 *Matlab Functions*

The volume V of liquid in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by

$$V = \left[r^2 \cos^{-1}\left(\frac{r-h}{h}\right) - (r-h)\sqrt{2rh - h^2} \right] L$$

Develop an M-file to create a plot of volume versus depth. Here are the first few lines:

```
function V = cylinder(r, L, h, plot_title)
% V = volume of horizontal cylinder
% inputs:
% r = radius
% L = length
% h = array representing different depth values
% plot_title = string holding plot title
```

For example, in the command window, you can the function as follows

```
>> cylinder(3,5, h_test, 'Volume versus depth for horizontal cylindrical tank')
```

where `h_test` is a user inputted array.
Use appropriate labeling for the plot.

Problem 3 *Taylor Series Analysis using MATLAB*

The Taylor series expansion for $\cos(x)$ is given as

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{2k!} \quad (|x| < \infty)$$

Write a MATLAB function that evaluates $\cos x$ (x is in degrees) using its Taylor series expansion and stores the output in the variable y. In order to evaluate the convergence of the Taylor series, your code should include a loop to track the value of the sum S_n where $S_n = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{2k!}$. At each iteration, the current sum S_n and previous sum S_{n-1} should used to calculate the relative error $E = \left| \frac{S_n - S_{n-1}}{S_{n-1}} \right|$. When E becomes smaller than 0.000001, the loop should be broken. (Hint: Use a while loop)
The function header is `[y,it]=EvalCos(x)` where the input `x` is in degrees, `y` is the calculated value and `it` is the number of iterations (terms) required to get the output.
Test your function for angles from 0^o to 360^o in increments of 15^o. Present the results in a tabular format. The table should contain five entries. The first corresponds to the input x, the second to the output y, the third to the exact value evaluated using the inbuilt function `cosd()`, the fourth to the number of terms needed and the last entry to the resulting absolute error as follows

```
x     y     yexact     iterations     Error
-------------------------------------------
```

Problem 4 *Taylor Series*

Show that the Mclaurin series of $\ln\left(\frac{1+x}{1-x}\right)$ is given as

$$\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + \cdots\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \quad |x| < 1$$

Using the same approach as described in class for $\ln(1+x)$ (Hint: $\ln(a/b) = \ln(a) - \ln(b)$)

Problem 5 *Alternating Series*

Verify that the following series fulfill the conditions of an alternating series. Then using the approach described in class, determine the number of terms that should be used to estimate the sum of the entire series with an error of less than 0.001. (Hint: use `www.wolframalpha.com` for the computations !!!!)

1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+3}$
2. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(\ln(n+3))}$

Comment on the obtained results.